BJT Amplifiers

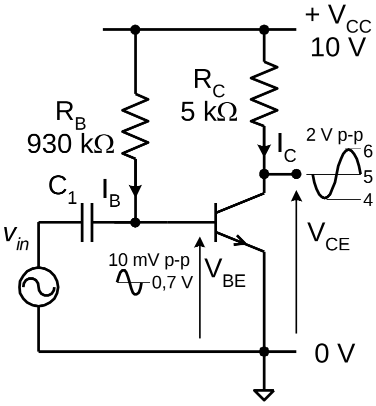
Text References:
Sedra & Smith: *Microelectronic Circuits*, Oxford University Press
Jaeger & Blalock: *Microelectronic Circuit Design*, McGraw-Hill

1. The BJT as an amplifier

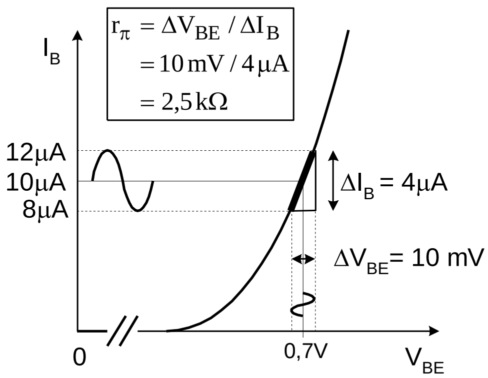
Consider a BJT with an operating or quiescent (Q) point placed centrally in the active region by a simple dc bias circuit as in Figure 1. Assume V_{BE} = 0,7 V and β = 100 giving the following dc bias conditions:

$$I_B = \frac{10-0{,}7\,V}{930k\Omega} = \underline{10\mu A}\,; \quad I_C = \beta \cdot I_B = 100 \cdot 10\mu A = \underline{1mA}\,; \quad V_{CE} = 10V - 1\,mA \cdot 5k\Omega = \underline{5V}$$

Now let a **small** ac input signal (10 mV pk-pk) be injected via a **coupling** capacitor (C_1) to the base and the output taken from the collector with the emitter grounded (common), giving **common-emitter** (CE) operation as in Figure 1. If capacitance C_1 is sufficiently large so that the reactance X_{C1} is small at the signal frequency, then $\Delta V_{BE} \approx v_{in}$ = 10 mV (pk-pk) as shown on the input characteristics in Figure 2.



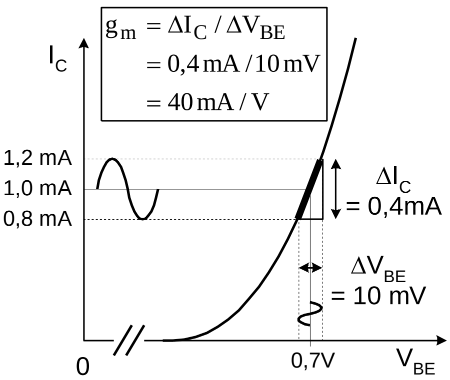
CE amplifier circuit
Figure 1.



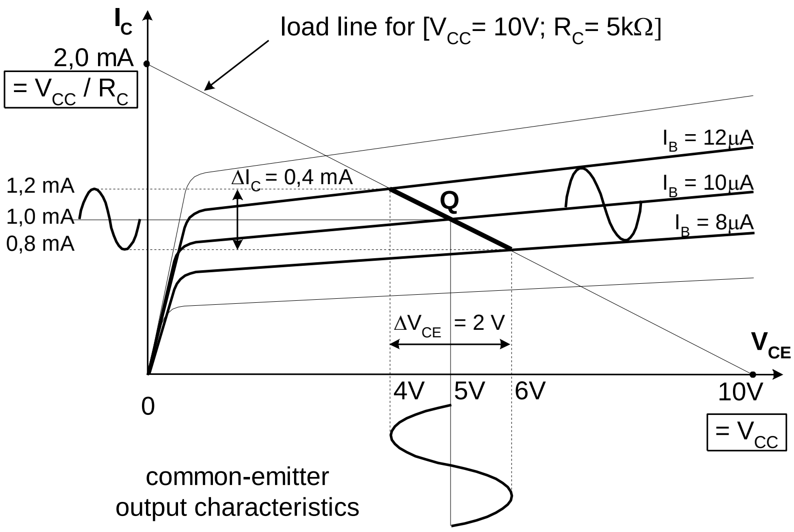
common-emitter
input characteristics

Figure 2.

The corresponding mutual and output characteristics and signals are shown in Figures 3 & 4.



common-emitter
mutual characteristics



common-emitter
output characteristics

Figure 3.

Figure 4.

As can be seen from the above figures, the **small signal** variations are **superimposed** upon the much larger **dc bias**. We may thus use the principle of superposition to analyse the circuit operation in two separate parts: the dc bias condition, and the small signal operation.

If the signals are sufficiently small, the curvature of the characteristics will be negligible and the circuit response will be approximately linear, so that a sinusoidal input voltage will produce an approximate sinusoidal output voltage as shown in the above figures. This condition is known as **small-signal operation** and allows us to use *linear* models to predict the amplifier performance.

From Figs. 2-4, assuming a linear response, the input signal voltage $v_{in} \approx \Delta V_{BE} = 10\,mV$ pk-pk gives a base current signal $\Delta I_B = 4\,\mu A$ pk-pk. Assuming linear operation $\Delta I_C = \beta \cdot \Delta I_B$ giving a collector current signal $\Delta I_C = 100 \cdot 4\,\mu A = 0{,}4\,mA$ pk-pk. Via the collector resistor $R_C = 5\ k\Omega$ this gives a corresponding output voltage signal $\Delta V_{CE} = 2\,V$ pk-pk, so this amplifier has a high signal voltage gain $A_v = \Delta V_{CE} / \Delta V_{BE} = 2\,V / 10\,mV = 200$ (inverting).

From the mutual characteristics (Fig. 3) the ratio of output current signal to input voltage signal is called the transconductance $g_m = \Delta I_C / \Delta V_{BE}$ ($= 0{,}4\,mA / 10\,mV = 40\,mA/V$).

From the input characteristics (Fig. 2) the BJT has a small-signal (dynamic) input resistance given by $r_\pi = \Delta V_{BE} / \Delta I_B$ ($= 10\,mV / 4\,\mu A = 2{,}5\,k\Omega$).

In practice these values are inconvenient to obtain graphically and the BJT exponential equation $I_C = I_S\, e^{V_{BE}/V_T}$ may be used to obtain these analytically as follows:

$$g_m \equiv \left.\frac{\partial I_C}{\partial V_{BE}}\right|_{Q\,pt} = \frac{1}{V_T} I_S\, e^{V_{BE}/V_T} = \frac{I_C}{V_T} \quad \text{and} \quad r_\pi \equiv \left.\frac{\partial V_{BE}}{\partial I_B}\right|_{Q\,pt} = \frac{\partial V_{BE} \cdot \beta}{\partial I_C} = \frac{\beta}{g_m} = \frac{\beta \cdot V_T}{I_C} = \frac{V_T}{I_B}$$

$$\text{Thus} \quad g_m = \frac{I_C}{V_T} = \frac{1\,mA}{25\,mV} = 40\,mA/V \quad \text{and} \quad r_\pi = \frac{\beta}{g_m} = \frac{100}{40\,mA/V} = 2{,}5\,k\Omega \ \text{as above.}$$

These may be used to give a simple **small-signal model** (known as the π model) for the BJT as shown in Fig. 5. The signal input voltage v_{be} appears across the dynamic input resistance r_π which carries base current i_b and the collector current is represented as a current source controlled by v_{be} (or i_b).

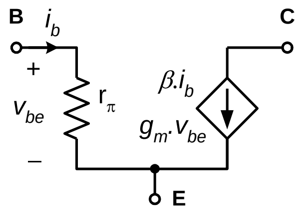


Figure 5

Since the output characteristics in Fig. 4 show dependence on V_{CE} due to the Early effect, the current source in Fig. 5 is not ideal and is better represented with finite output resistance r_o where

$$r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C} \quad \text{(see Fig. 6 later).}$$

This and other small-signal models for the BJT are outlined below.

2. Small-signal BJT models (for low and medium frequencies)

For low and medium frequency analysis of BJT amplifier circuits, the following small-signal BJT models are commonly used:

2.1 The π model:

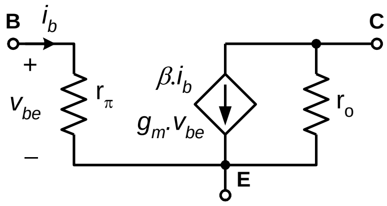


Figure 6. The π model

Where: $$g_m = \frac{I_C}{V_T}; \quad r_\pi = \frac{V_T}{I_B} = \frac{\beta \cdot V_T}{I_C} = \frac{\beta}{g_m} = (\beta+1)\cdot r_e; \quad r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

In cases where r_o is much greater than the effective load resistance, r_o may often be neglected, further simplifying the model. This model is convenient to use when the emitter is grounded. This is a simplified form of what is known as the Hybrid-π model of a BJT.

2.2 The T model:

This is similar to the π model but reflects the dynamic input resistance into the emitter path as r_e so v_{be} appears across the much smaller resistance r_e carrying the much larger emitter current i_e. Thus r_e represents the dynamic input resistance looking into the emitter and hence $r_e \equiv \left.\frac{\partial V_{BE}}{\partial I_E}\right|_{Qpt} = \frac{\partial V_{BE} \cdot \alpha}{\partial I_C} = \frac{\alpha}{g_m} = \frac{\alpha \cdot V_T}{I_C} = \frac{V_T}{I_E}$ and comparing with r_π, we find $r_e = \frac{r_\pi}{\beta+1}$.

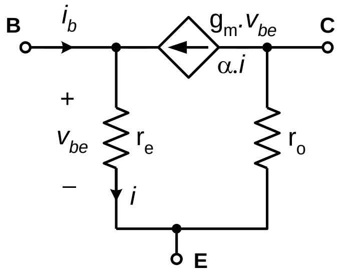


Figure 7. The T model

Where: $$g_m = \frac{I_C}{V_T}; \quad r_e = \frac{V_T}{I_E} = \frac{\alpha \cdot V_T}{I_C} = \frac{\alpha}{g_m} = \frac{r_\pi}{\beta+1}; \quad r_o = \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

Again, in cases where r_o is much greater than the effective load resistance, r_o may often be neglected, further simplifying the model. This model is convenient to use when there is resistance in the external emitter circuit.

2.3 The common emitter *h*-parameter model:

The *h*-parameter model values are often given in data sheets for low to medium frequency BJTs. This model is based on a set of two-port parameters related by the following equations:

$$v_{be} = h_{ie} i_b + h_{re} v_{ce}$$
$$i_c = h_{fe} i_b + h_{oe} v_{ce}$$

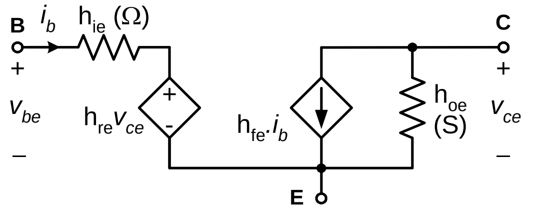


Figure 8. The full *h*-parameter equivalent circuit

The full *h*-parameter small-signal equivalent circuit is given in Fig. 8.

Usually h_{re} is negligibly small yielding the **simplified** *h*-parameter model in Fig. 9. Comparison with Fig. 6 indicates this is identical with the π model with:

$$\boxed{r_\pi = h_{ie}\ ; \quad \beta = h_{fe}\ ; \quad r_o = 1/h_{oe}}$$

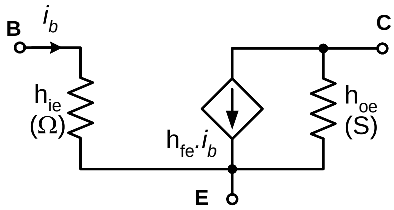


Figure 9. Simplified *h*-parameter model

2.4 BJT models: simple comparison/summary

These BJT models may be simply summarised using the "resistance reflection" principle shown in Fig. 10 below. To simplify the comparison, neglect r_o. (It may be simply added between the C and E terminals in any of the models if required.) The π-model of Fig. 6 may then be redrawn as shown in Fig. 10 (a) and the T-model of Fig. 7 may be redrawn as in Fig. 10 (b) below.

Inspection shows that the only difference between these two models is shifting the resistances r_π and r_e between the base and emitter circuits using the "resistance reflection" principle.

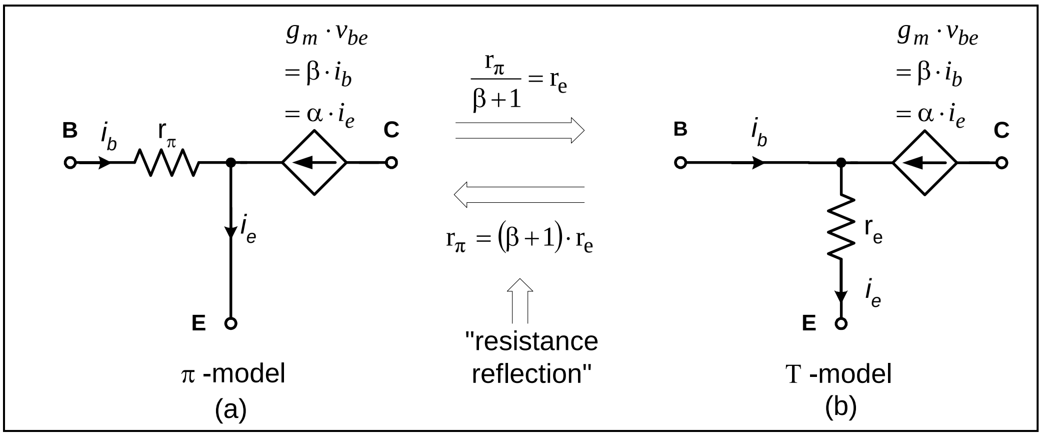


Figure 10

These models may be regarded as equivalent to the transistor **as far as *small-signal* operation** is concerned and may thus be used to develop a small-signal equivalent circuit of the amplifier. To determine the model values the dc bias condition must first be established. Note that these models only represent operation at low and medium frequencies, and more elaborate models will be used to determine high frequency behaviour. This will be covered in the Analogue Electronics 2 course.

3. Mid band single-stage BJT amplifier analysis

3.1 Common emitter (CE) amplifier

The common emitter stage is probably the most frequently used stage as it exhibits characteristics suitable for most general applications. It offers both high voltage and current gain hence the largest power gain with moderate input and output resistances.

Consider the CE stage shown in Fig. 11. Here a typical four-resistor bias circuit is used to establish the chosen dc bias conditions (Q point). The ac signals are then coupled into and out of this circuit by means of the **coupling** capacitors C_1 and C_2 which also ensure that the dc bias conditions are unaffected by the external connections i.e. **dc blocking**. The emitter is grounded for signals by the **bypass** capacitor C_E hence the signal connection is input to the base, output from the collector with the emitter common (grounded) hence *common emitter*.

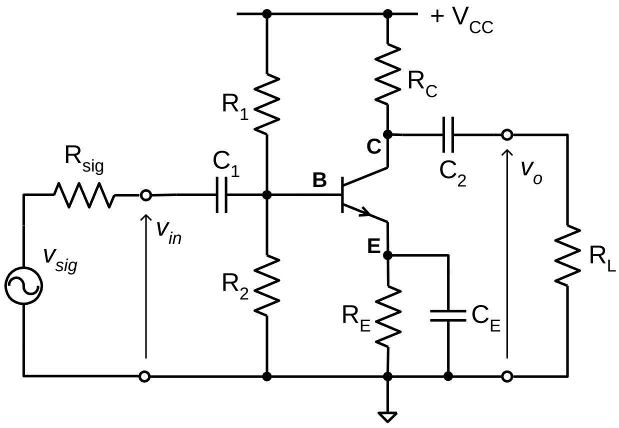


Figure 11. Common emitter stage

At so-called **mid-band** frequencies the coupling and bypass action of the capacitors may be treated as ideal (i.e. their reactances regarded as negligibly small) and the high frequency limitations of the BJT may be neglected.

The small-signal analysis of such BJT amplifier circuits is carried out as follows:

1. Taking the coupling and bypass capacitors as open circuits (dc blocking), obtain the dc bias circuit and hence calculate the dc bias conditions (in particular I_C or I_E).

2. Obtain the small-signal model values for the BJT using the dc bias information.

3. Draw a **small-signal (ac) equivalent circuit** for the amplifier by taking all coupling and bypass capacitors as short circuits, treating dc power supplies as *signal* grounds, and replacing the BJT by its small-signal model. The small-signal analysis of the amplifier then follows simply from this equivalent circuit.

NOTE 1: In the small-signal equivalent circuit there are **no dc quantities**. (e.g. $v_{be} \neq 0{,}7\,\mathrm{V}\,!$) All voltages and currents are **signals**.

NOTE 2: The BJT small-signal models are **the same for *npn* and *pnp* BJTs**. The signal current directions in Figs. 6 – 10 are the used for both *npn* and *pnp* BJTs.

To illustrate the simplicity of step 3 (drawing the small-signal equivalent circuit), consider doing it in stages for the circuit in Fig. 11. Treating the coupling and bypass capacitors as short circuits and the dc power supply as signal ground we obtain Fig. 12 which may then be simply redrawn as in Fig. 13 from which the common-emitter connection is obvious.

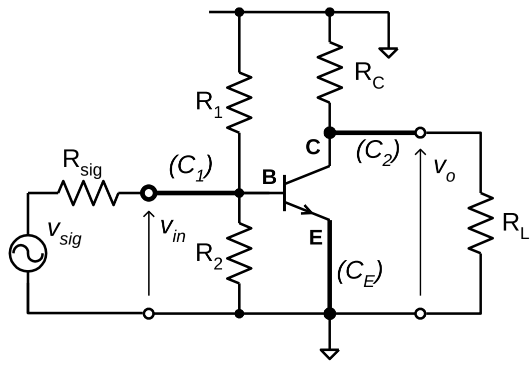


Figure 12.

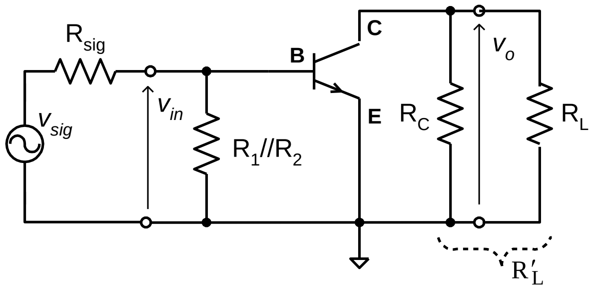


Figure 13.

Replacing the BJT with a small-signal model (e.g. π model) we then simply obtain the final small-signal equivalent circuit of the CE amplifier as shown in Fig. 14. The signal analysis then follows directly from this equivalent circuit.

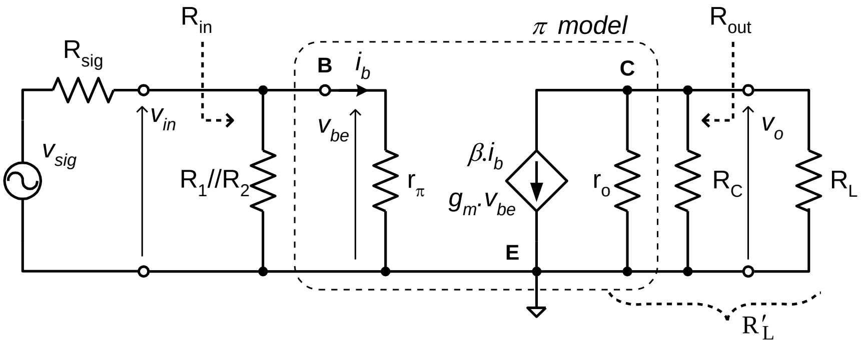


Figure 14. Small-signal equivalent circuit for CE stage

The input and output resistances are: $R_{in} = R_1 // R_2 // r_\pi$; $R_{out} = R_C // r_o$

Noting that $R'_L = r_o // R_C // R_{L_o}$, the voltage gain at the amplifier terminals is found as follows:

$$A_v \equiv \frac{v_o}{v_{in}} = \frac{(-g_m v_{be}) \cdot R'_L}{v_{be}} = -g_m R'_L \quad \text{or}$$

$$A_v = \frac{(-\beta \cdot i_b) \cdot R'_L}{i_b \cdot r_\pi} = -\frac{\beta R'_L}{r_\pi}$$

The CE amplifier is inverting as indicated by the negative sign of the voltage gain.
If the overall voltage gain referred to the source emf is required, then the voltage divider at the input will have to be taken into account as follows:

$$A_{v_{sig}} \equiv \frac{v_o}{v_{sig}} = \frac{v_o}{v_{in}} \cdot \frac{v_{in}}{v_{sig}} = -g_m R'_L \cdot \left(\frac{R_{in}}{R_{in} + R_{sig}} \right)$$

Note: *Sedra & Smith* uses G_v for $A_{v\,sig}$;
Jaeger & Blalock uses A_{vt} for A_v and A_v for $A_{v\,sig}$

3.2 Common emitter amplifier with emitter resistance (CE with R_E)

The emitter resistance (or a portion) in the common emitter stage may be left unbypassed to reduce the voltage gain and increase the input resistance. The reduced voltage gain then becomes more determined by passive resistor ratios than on the BJT properties, a benefit of negative feedback.

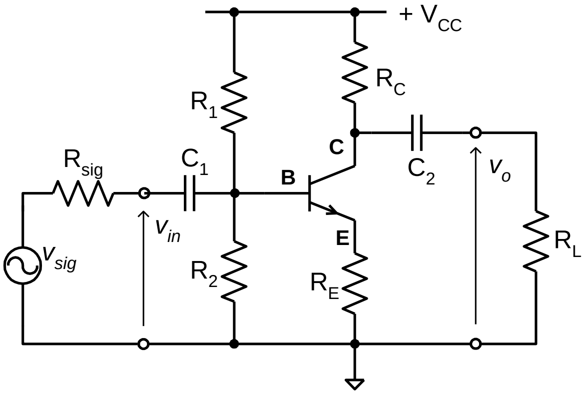


Figure 15. Common emitter stage with emitter resistance

Analysis is convenient using a T model for the BJT, and is further simplified by neglecting r_o which is often realistic since usually $r_o >> R'_L$.

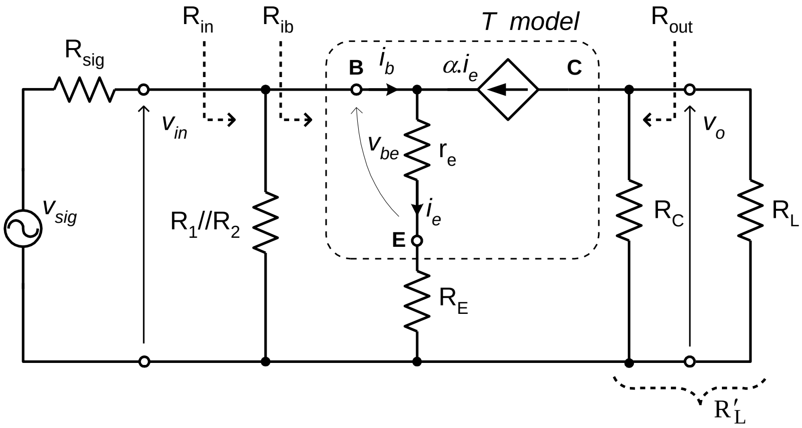


Figure 16. Small-signal equivalent circuit for CE stage with R_E (using T model)

$$R_{ib} = \frac{v_{in}}{i_b} = \frac{i_e(r_e + R_E)}{i_b} = \frac{(\beta+1)\, i_b (r_e + R_E)}{i_b} = (\beta+1)(r_e + R_E)$$

$$R_{in} = R_1//R_2//R_{ib} \; ; \qquad R_{out} = R_C$$

$$A_v \equiv \frac{v_o}{v_{in}} = \frac{(-\alpha \cdot i_e)\, R'_L}{i_e(r_e + R_E)} = -\alpha \frac{R'_L}{r_e + R_E} \cong -\frac{R'_L}{r_e + R_E}$$

$$A_{v\,sig} \equiv \frac{v_o}{v_{sig}} = \frac{v_o}{v_{in}} \cdot \frac{v_{in}}{v_{sig}} = A_v \cdot \left(\frac{R_{in}}{R_{in} + R_{sig}} \right)$$

Note the "resistance reflection" property evident in the equation for R_{ib}.

Alternatively $$A_v = -\alpha \frac{R'_L}{r_e + R_E} = -\frac{\beta}{(\beta+1)} \cdot \frac{R'_L}{(r_e + R_E)} = -\frac{\beta R'_L}{r_\pi + (\beta+1) R_E} = -\frac{\beta R'_L}{R_{ib}}$$

This version is obvious if the π model is used as shown in Figure 17. (*Do this as an exercise*).

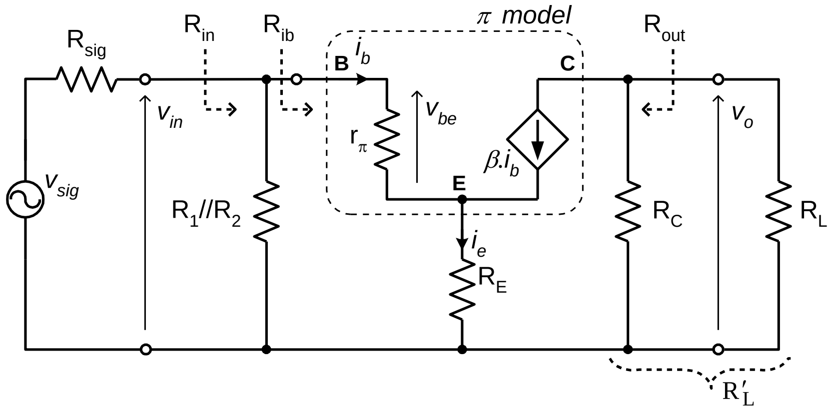


Figure 17. Small-signal equivalent circuit for CE stage with R_E (using π model)

3.3 Common base (CB) amplifier

The common base stage is non-inverting and has a very low input resistance. Its main advantage is its wide bandwidth and it is most commonly used combined with a common emitter stage in a combination known as the cascode circuit (which will be studied in Analogue Electronics 2).

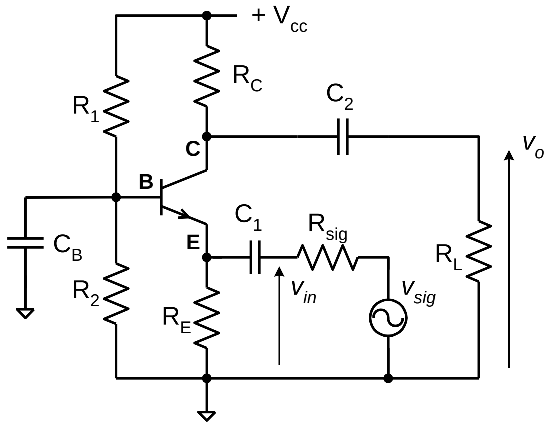


Figure 18. Common base stage

Analysis is convenient using a T model for the BJT further simplified by neglecting r_o as shown in Figure 19 below.

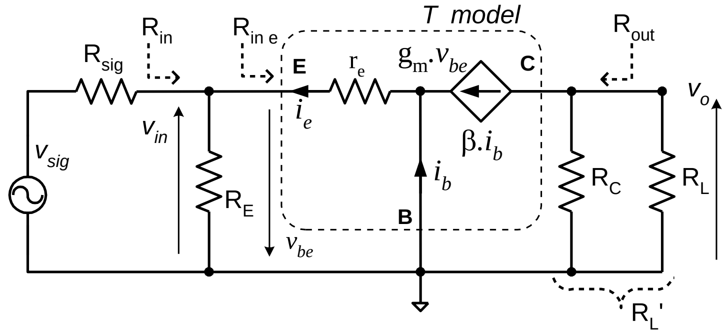


Figure 19. Small-signal equivalent circuit for CB stage

$$v_o = -g_m \cdot v_{be} \cdot R'_L \qquad \text{and} \qquad v_{in} = -v_{be}$$

$$A_v \equiv \frac{v_o}{v_{in}} = \frac{-g_m \cdot v_{be} \cdot R'_L}{-v_{be}} = +g_m \cdot R'_L \qquad \therefore \quad A_{vsig} \equiv \frac{v_o}{v_{sig}} = +g_m \cdot R'_L \cdot \frac{R_{in}}{R_{in} + R_{sig}}$$

$$R_{in\,e} = r_e = \frac{r_\pi}{\beta + 1} \approx \frac{1}{g_m} \qquad \therefore \; R_{in} = R_E // r_e \, ; \qquad R_{out} = R_c$$

The voltage gain is thus seen to be the same as for the common emitter stage, except non-inverting, but the input resistance is significantly reduced (by about β+1 times).

3.4 Common collector (emitter follower) amplifier (CC or EF)

The common collector or emitter follower stage is popularly used as a voltage buffer to give a high input resistance at the input to a circuit or a low output resistance at the output of a circuit. Like the common base stage it is also non-inverting, and has a very wide bandwidth. Since the output is now the emitter, dc biasing would usually be arranged to allow maximum output voltage swing at the emitter.

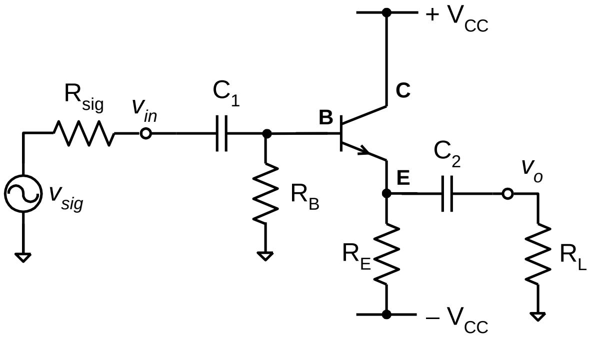


Figure 20. Common collector or emitter follower stage

Small signal analysis is convenient using a T model for the BJT as shown in Figs. 21 and 22. Substituting the T model leads directly to Fig. 21(a).

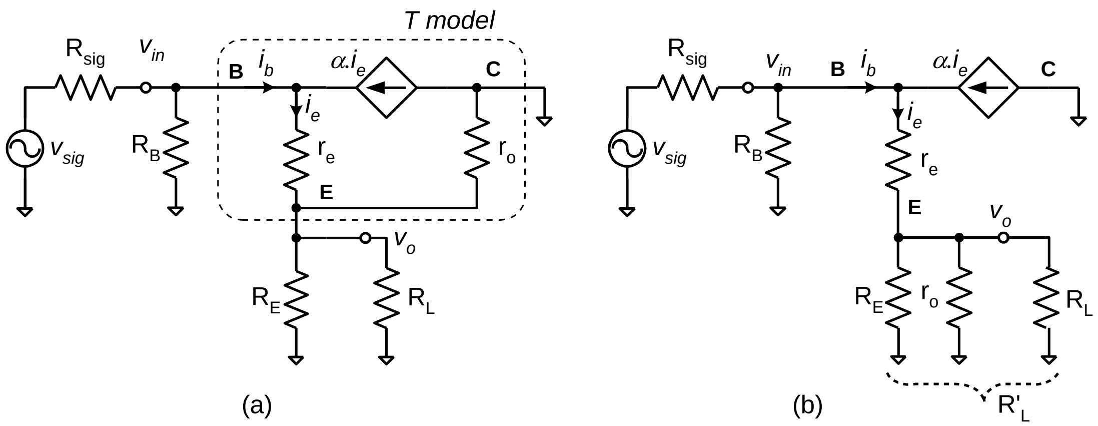


Figure 21.

Effectively r_o appears across the output as shown in Fig. 21(b) hence $R'_L = R_E // r_o // R_L$
Fig. 21(b) above may be redrawn as in Fig. 22 below.

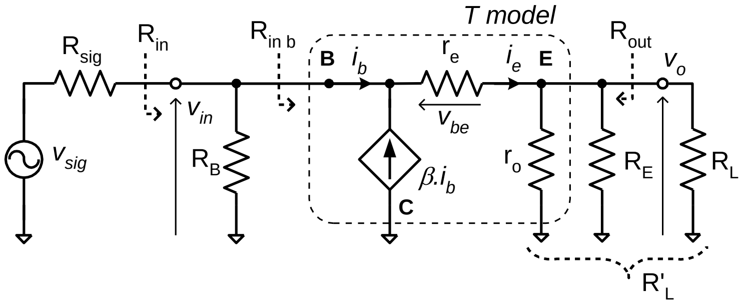


Figure 22. Small-signal equivalent circuit of CC (EF) stage (using T model)

$$R_{in\ b} \equiv \frac{v_{in}}{i_b} = (\beta+1)(r_e + R'_L)\ ; \quad R_{in} = R_B // R_{in\ b}\ ; \quad R_{out} = R_E // r_o // \left(r_e + \frac{R_{sig} // R_B}{\beta+1} \right)$$

$$A_V \equiv \frac{v_o}{v_{in}} = \frac{R'_L}{r_e + R'_L} \quad \left(\cong 1 \ \text{ if } \ r_e << R'_L \right)$$

$$A_{V\,sig} \equiv \frac{v_o}{v_{sig}} = A_V \cdot \frac{R_{in}}{R_{in} + R_{sig}} = \frac{R'_L}{r_e + R'_L} \cdot \frac{R_{in}}{R_{in} + R_{sig}}$$

Considering typical numerical values, it can be seen that this circuit yields a very high input resistance, a very low output resistance, and almost unity voltage gain, as desired for a voltage buffer stage.

(Note: The expression for output resistance above is easily seen from the circuit diagram using "resistance reflection")

Alternatively, using the π model instead for the BJT simply yields Fig. 23.

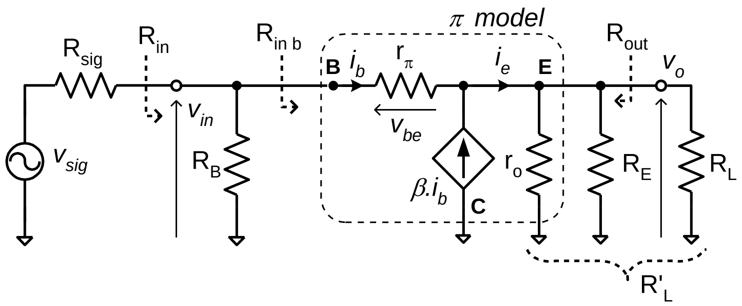


Figure 23. Small-signal equivalent circuit of CC (EF) stage (using π model)

From Fig. 23 we easily obtain:

$$A_V \equiv \frac{v_o}{v_{in}} = \frac{v_o}{i_b r_\pi + v_o} = \frac{(\beta+1)\, i_b R'_L}{i_b r_\pi + (\beta+1)\, i_b R'_L} = \frac{(\beta+1) R'_L}{r_\pi + (\beta+1) R'_L} \quad \left(= \frac{R'_L}{\frac{r_\pi}{(\beta+1)} + R'_L} = \frac{R'_L}{r_e + R'_L} \right)$$

(as obtained from Fig. 22 above)

These single amplifier stages may be combined into a multistage amplifier to produce higher gains and/or to give particular input and output resistances (see Tutorial AN5).